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...I believe that economics and business would be more productive fields if we learned something important from evolutionists: that models are metaphors, and that we should use them, not the other way around.

Paul Krugman, addressing European Association for Evolutionary Political Economy in 1996.

Discovering Patterns and Models in Economic and Business Analysis

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Introduction

The paper attempts to make a series of reflections regarding the role of modeling and conceptualization in the process of economic and business reasoning. The essay headline proposes that most of existing economic models are heavily influenced by artificial thinking instruments and do not represent economic reality adequately. Neoclassical concept of the market emphasizes the role of a priori assumptions and does not see it as a place for learning and testing reality; thus its practical applicability is limited in scope at least. This has become unquestionable with the arrival of so called "new economy" - existing economic models can not explain new emerging patterns of the market and business strategies. Well-known problem of the bar El Farol is used to show forgotten features of the market (market ecology, role of human expectations, internal dynamics, agent-based modeling). Finally, using a simple example, I will introduce Partitioning and Tearing Method, and I will advocate the use of new approach in economic and business thinking, based on the Systems Dynamics, visual thinking, and computer supported modeling and simulation.

1. Levels and stages in economic thought

Understanding empirical phenomena is perhaps the most important task for economic science. Unfortunately, as theoretical studies and practical actions have shown, it has been failing in both, micro (enterprise) and macro (market) level. Numerous and mathematically advanced economic analysis, almost impenetrable for practitioners, do not provide a sufficient level of analytical insight for making decisions and defining growth policies and business strategy. It has become more and more clear that new approaches are required and new methodologies are waiting to be discovered/invented. That is basically the problem of reckoning economic complexity and finding suitable instruments that could tackle with complex phenomena in existing markets.

Stephen N. Durlauf (2002) wrote that complexity in economic analysis has been handled in three complementary ways. Historical study is first of them; if there is any rational explanation of economic processes, it should be provable in historic analysis. That would be feasible, if economic processes were the result of one, supreme logics and a common mechanism. What happens, however, if such a mechanism and logics does not exist and the production and distribution is governed by many simultaneously operating and different logics?

The identification of certain statistical properties in socio-economic populations has been another target in the analysis of complex economic systems. It is commonly referred to as Power and Scaling Laws. In general, a random variable is said to obey the Power Law if it has an associated distribution function following Pareto's model (e. g. Axtell 2001 on company's sizes). Those studies are usually conducted in connection with the Scaling Law where changes in variables existing in a given scale (e. g. firm size) lead to linked changes in other associated variables existing in different dimension (e. g. profit). Typically, the Laws of Power and Scaling are found in linear analysis.

Third way of dealing with the complexity in economic systems is the analysis of social interactions, in particular individual and social behavior in economic context. Dominant studies have attempted here to construct probability models enabling to predict behavior from a given set of assumptions on individual and group level variables. This is a critical problem in economic science - the questions if the market is a self-containing entity or the sum of individual behaviors still remain open; so does the problem of required market variables/agents aggregation.

None of these approaches has been proved successful in practical analysis. It seems that increasing sophistication of mathematical modeling in economic analysis is more connected to mathematical tools themselves than to the contents of the market problem. There is an opinion that the development of mathematics has evolved from the concept of "place" to "pace" and to "pattern". "Place" refers to Euclidean geometry in the ancient world. "Pace" refers to the discovery of calculus by Newton and Leibniz in the 17th century along with linearity as a proof of ultimate scientific correctness. Both have marked dominant paths in analysis, emphasizing locality and present time in economic thought. It has been accompanied by traditional research methodology where hypothesis testing and results formalization (theory creation) is a common pattern. Moreover, increasing formalization of economic models has reached such a point that ontological assumptions underlying economic models are mathematical models themselves (Sergeyew 1997). Vital connections between real life economic phenomena and theoretical thought have been cut off and, gradually, lost or replaced by other methodological approaches. Dogs are wagging owners...

"Pattern" concept is accessible through current terminology in science; many buzzwords and phrases like chaos, complexity, emergent properties, and self-organizing systems are only few examples. We can interpret a pattern as an internal order/structure of a problem that equally (author's underline) depends on modeled reality as on a person conducting analysis. A pattern, therefore, is a set of assumptions (frequently not articulated explicitly) concerning internal relationships among linked variables - components of the problem under analysis. An interesting feature of those "patterns" is that they are grounded not in old but new sets of assumptions rather; they possess clearly different ontological foundation which is frequently undefined, fuzzy, and intuitive. That new emphasis on pattern (as distinct from formal hypothesis-testing using simple refutable statements to be tested with data) calls for a metaphor and mental models and make them an increasingly prominent instrument in economic science. Place, pace, metaphors, and mental models of patterns are recognizable levels (stages) in economic and business thinking.

2. Complex, adaptive, non-linear economic systems

Thus, metaphors are recent substitutes for formal mathematical tools. Metaphors are implicit statements that hold in tension two incompatible or opposite meanings which reveal a new insight into the analysis. The metaphors and mental models allow discussing about yet undefined concepts, thus they stimulate new thinking. For instance, a basic (mathematical) metaphor for the neo-classical model of the market has been the construct of mechanical equilibrium according to which even small deviations of the market from the point of equilibrium produce counteracting forces regaining the equilibrium state.

The metaphor of mechanical equilibrium and whole neo-classical economics confines economic analysis to rational framework, within which all economic agents (producers and customers) try to make sense of available information and attempt to act accordingly. Thus, that specific metaphor enforces equally specific cognitive pattern for the market:

> Economic agents are born rational and deprived of emotions.

> They constantly assess existing uncertainty.

> They try to reduce statistically the uncertainty, using formal reasoning and on-going information analysis (e. g. Bayesian theorem).

> At a certain point of time, they feel sufficiently confident to choose a course of action that maximizes their expected utility.

> All acting agents meet in a impersonal setting (market) where they can only see results of decisions made by other agents, thus:

> They do have a similar amount of knowledge and information about each other, and:

> They do not learn from each other.

Neo-classical concept of the market is based then on a very simple set of assumptions. The market is not a space in which agents can reduce the uncertainty in interactive way but the place for maximizing local rationalities. The sum of local rationalities creates a hierarchical structure on the top of which we find the state described as the market equilibrium. The equilibrium state, as well as any deviation from it, is immediately detected by acting agents, and - as they can not learn from each other - it stabilizes cognitive structure of the market existing in their minds.

What happens, however, if that simple set of cognitive assumptions does not work and the market follows a different pattern (W. Bryan Arthur 1997)? Set of opposite assumptions would be a viable alternative for the neoclassical concept of the market and economy. Accordingly we find here:

> Instead of many independent players we have interactions of a large number of dispersed and heterogeneous agents acting in parallel. The action of any given agent depends upon the anticipated actions of a limited number of other agents and on the aggregate state these agents co-create.

> Market does not control agents' behavior. Controls are provided by mechanisms of competition and coordination between agents, part of which is consciously undertaken and another part does not depend on anyone.

> Market has many levels of organization and interaction. Instead of linear hierarchy we have a complex construction of behaviors, actions, strategies, and products that typically serve as `building blocks' for making up units at the next higher level. The

overall market organization is more than hierarchical, with many sorts of crossing and entangling interactions.

> Behaviors, actions, strategies, and products are revised continually as the individual agents accumulate experience – the market constantly adapts to individual learning processes occurring in it.

> Constant adaptation is based on experimentation and novel behaviors. Complex structure of new markets, new technologies, new behaviors, and new institutions creates the pressure toward ongoing, perpetual novelty.

> Interactions, competition/coordination, complex structure, and constant improvements create the state where mechanical equilibrium can not endure. Instead, there is a stable market in changing environment. W. Bryan Arthur uses the term "out-of-equilibrium dynamics" here; market stability stems from its dynamic changes.

3. Expectational economy

Neoclassical notion of the market and economy is oriented towards outcomes. Its counter-alternative emphasizes processes that yield those outcomes. They differ in the sense that the former sees economic agents' behavior as subject to the state of equilibrium, and the latter considers deviations caused by them as the source of market dynamics, determining – in a long run – market stability and survival. The difference is not only semantic – the term "market dynamics" is a key concept here. Neoclassical market contains a negative feedback mechanism; interactive and agent-based market (non-linear economic systems) uses positive feedback. The concept of increasing return, impossible in neoclassical economy and unseen in traditional markets, has become reality.

W. Bryan Arthur is probably the best known defender of the increasing return economy. He proposes to describe that new economic reality as "expectational economy" – the economy where processes and not outcomes play the most important role, negative feedback is sometimes suppressed by positive feedback mechanisms, and where processes and outcomes depend on individual learning, expectations, and interactions.

W. Bryan Arthur's El Farol Bar Problem is my favorite example here (W. Bryan Arthur 1999). El Farol is a cozy bar in Santa Fe (New Mexico), the town hosting famous Santa Fe Institute. El Farol is a popular place to visit for local bohemia and scientists who want to listen to Irish guitar music or Mexican Mariachi. One hundred of visitors decide independently each weekend whether to show up at their favorite bar or to stay home. The rule is that if a person predicts that more that 60 (arbitrary value) will attend, he or she will try to avoid the crowds and stay home; if the prediction is that fewer than 60 will be in the bar, he/she will go. The main question is: how the bar-goers each week might predict the numbers of people showing up in El Farol, and what is the resulting dynamics of El Farol attendance.

There are two features of the El Farol problem. First, it will become quickly obvious to the agents that predictions of how many will attend the bar depend on others' predictions of how many attend (attendance determination). Similarly, others' predictions in turn depend on their predictions of others' predictions. If agents use only their deductive capabilities, there is an infinite regress and no rational decision can be made. No "rational" set of expectations can be assumed as agents' common knowledge, and from their viewpoint, the problem is ill-defined. Second – any commonality of expectations gets broken up: if all expect that few will go, all will go, invalidating initial assumption.

Similarly, if all believe that more than 60 will go, nobody will go, thus invalidating that belief.

In 1993 W. Bryan Arthur (who actually designed and conducted El Farol experiment) modeled this situation by assuming that while the agents visit the bar, they act inductively-they act as conscious statisticians, each starting with a variety of subjectively chosen expectational models or forecasting hypotheses. Each week they act on their currently most accurate model. Thus agents' beliefs or hypotheses compete for use in an ecology these beliefs create. Computer simulation (Fig. 1) shows that the mean attendance quickly converges to 60. In fact, the predictors self-organize the population into an equilibrium "ecology", in which of the active predictors 40% on average are forecasting above 60 and 60% below 60. This emergent ecology is organic and self-organizing in nature.

Interestingly, if we change active proportion from 60/40 to, say, 70/30 or 90/10, emerging structure will show exactly the same properties. After a while the mean attendance will follow initial hypothesis and expectation and converge to 70 and 90, respectively. Another interesting point - attendance adjustment to individual expectations does not mean that a fixed set of customers will be attending El Farol. Different persons will be in the bar within the same evolving ecological structure which has been proved by empirical observation and computer simulation.

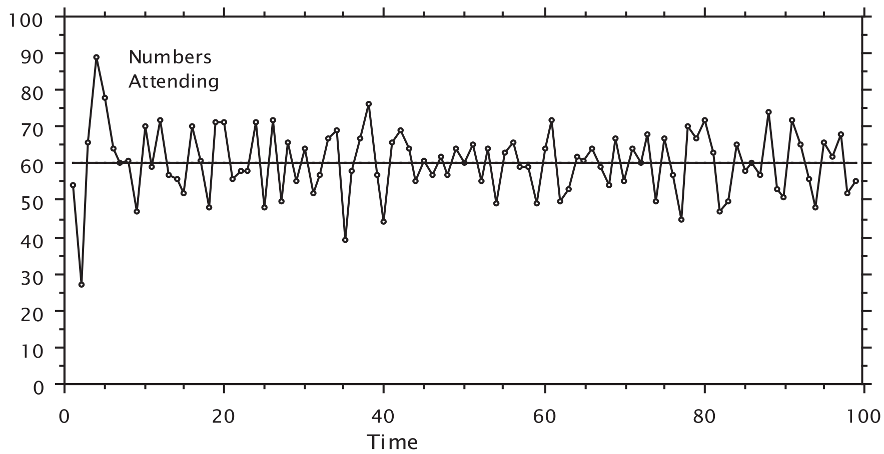


Fig. 1. El Farol Attendance

Source: W. Bryan Arthur 1999.

4. Discovering patterns in complex, economic systems

El Farol is a miniature of expectational market and economy. It is an apparently simple problem with highly complex behavior that can not be explained by standard economics. More often those complex, agent-based entities are ever-changing, showing perpetually novel behavior and emergent phenomena. Complexity, therefore, portrays the economy not as deterministic, predictable and mechanistic but as process-dependent, organic and always evolving. As such, there is no rational prediction and predetermined state does not exist. Instead, continually learnt behaviors and modified expectation become market driving forces that govern the market. Agents must acquire the knowledge about those driving forces unless they "do not care about El Farol" (disappear from the market). Standard theories of financial markets assume rational expectations-that investors adopt uniform forecasting models which are on average validated by forecasted prices. The theory works well at the beginning only; market "anomalies" such as unexpected price changes, sudden price variation, and demand changes caused by speculation remain out of range for these theories. Holland, LeBaron, Palmer, and W. Bryan Arthur (1997) have created a model which relaxes rational expectations by assuming, as in the El Farol problem, that investors can not assume or deduce expectations but must discover them (underline by author). Our agents continually create and use multiple "market hypotheses"-individual, subjective, expectational models-of future prices and dividends within an artificial stock market on the computer. The result was a self-contained, artificial financial world, where - like the bar El Farol - it is a "mini-ecology" in which expectations compete in a world these expectations create. Can metaphors and mental models create the reality?

Fifty years ago R. Merton described such a situation and introduced the term "self-fulfilling prophecy". The market works as a self-fulfilling prophecy; agents have (formulate) some assumptions at the beginning and their actions are compliant with those assumptions. What happens then, however, has nothing to do with assumptions; actions (even if based on false premises) lead to results confirming them and further actions depend less and less on initial assumptions and more and more on the learning process. When problems are too complicated or when they are not well-specified, agents face not a problem but a situation. They must deal with that situation; they must frame the problem, and that framing in many ways is the most important part of the decision process. Therefore, what lies between the problem and the action is human cognition. Agents work with mental models, metaphors, and "selective abstractions" of the reality in which they act. They learn and reason; they can apply many different ways of making inference about the population, test them, make decisions, and act. A mental model of their reality will have, thus, utmost importance.

Is it possible to predict the behavior of El Farol customers? W. Bryan Arthur's work have proved that described situation is predictable not in terms of statistical decision theory but as the result of an on-going learning and modifications of previous mental model. Moreover, such a prediction is not only possible for an external expert advisor - it is equally available for El Farol customers. Although most known formal simulations of this problem were made in SWARM-C computational environment (simulation software elaborated in Santa Fe Institute), interested agent can do the same. Any market agent is able to rationalize decisions and acts using some methodological and technical support for understanding a pattern underlying market reality.

5. Problems and complexities

System dynamics is a methodology developed for studying and managing complex feedback systems, such as one finds in business and other social systems. The concept had been developed by Professor Jay W. Forrester at Massachusetts Institute of Technology in the early 1960s. At that time, he began applying what he had learned about systems during his work in electrical engineering to everyday kinds of systems. Traditional analysis focuses on the separating the individual pieces of what is being studied; in fact, the word "analysis" actually comes from the root meaning "to break into constituent parts". Systems thinking, in contrast, focuses on how the thing being studied interacts with the other constituents of the system-a set of elements that interact to produce behaviour-of which it is a part. Therefore instead of isolating smaller and smaller parts of a system (local rationalities), systems thinking involves a broader view, looking at larger and larger numbers of interactions. Once a pattern underlying defining interactions is detected, systemic structure of the problem under analysis is found. Therefore, understanding behavior without the structure is impossible and modifying structure of the problem is the only way of solving it; solution is the intervention into the structure. From that standpoint, System Dynamics is the art and science of making reliable inferences about behavior of a problem by developing an increasingly deep understanding of structures underlying that behavior.

The dependence of behavior upon structure is the core concept of the theory of autopoietic systems (Varela, Maturama 1987). Described sometimes as the property of recursiveness, the theory states that components of the problem participate recursively in the same network of relationships that produced them, and they realize the network of relationships as a unity in the space in which the components exist.Autopoiesis is a process whereby a system produces its own organization and maintains and constitutes itself in a space. Autopoiesis is the archetype of pattern. The secret of understanding El Farol behavior lies in determining its autopoiesis.

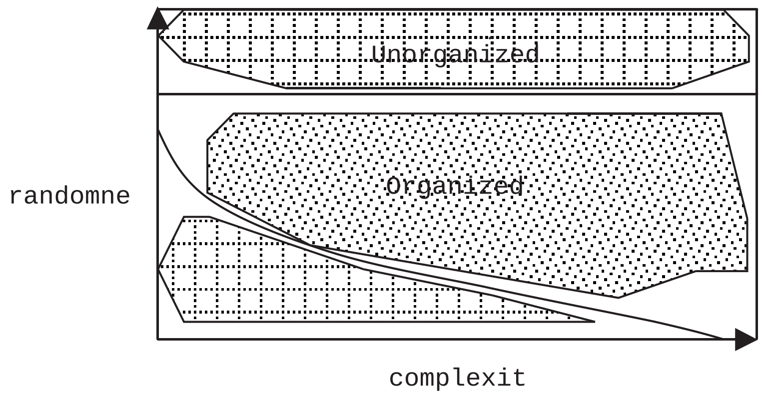
One may ask how it is possible that such a simple situation as the one described in El Farol case can produce such a complex behavior. The answer lies in the concept of complexity - the construct explaining even most complicated behavior patterns. Oryginal root meaning of the term (from Latin "complexus") signifies "entwined", "twisted together". This may be interpreted in the following way: in order to have a complex we need two or more components, which are joined in such a way that it is difficult to separate them. Similarly, the Oxford Dictionary defines something as "complex" if it is "made of closely connected parts". Intuitively, a system would be more complex if more parts could be distinguished, and if more connections between them existed.

Those two aspects of the complexity - distinction and connection - determine two dimensions characterizing complexity. Distinction corresponds to variety, to heterogeneity, to the fact that different parts of the complex behave differently. Connection corresponds to the connectivity where behavior of one part cannot be explained and predicted without a functional reference to the behavior of other connected parts.

Thus, we have two types of complexity; if there are many variables composing a whole we use the term of "detail complexity". Most people think of complexity in terms of the number of components in a system or the number of combinations one must consider in making a decision. But there is another type of complexity. The second type is "dynamic complexity" - situations where cause and effect are subtle, and where the effects of interventions are not obvious over time. Conventional tools and analysis methods are not equipped to deal with dynamic complexity as many complex and behavior patterns arise from a system composed of few parts (as, for instance, the structure of El Farol presented further). Dynamic complexity arises from the interactions of the problem components over time (np. Sterman 2000, p. 21). In the case of El Farol detail complexity is very low; eventually all bar customers can be divided into two groups - those who go and those who do not.

It would be interesting to classify problems according to their complexity dimensions. Two aspects could be used here - type and change in complexity and predictability/randomness of behavior. The combination of both provide us with three areas picturing different problem categories along with different ways of thinking and solving them (see: Fig. 2).

Fig. 2. Types of problems



Source: author's elaboration based on: Weinberg 1975, p. 18

Organized complexity contains mechanistic problems where they can be disassembled and re-assembled again without loosing functional character of their parts. For those problems we can use analytic approach and, typically, problem is decomposed into a number of indepent sub-problems which, in turn, can be solved witout affecting the correctness of the whole problem (machines and engineered systems are typical examples). Unorganized complexity is the universe of problems containing very large number of elements, behavior of which - even if random - can be translated into a mathematical or statistical equation. Statistical populations (sometimes called "aggregates") are usual examples of unorganized complexities (e. g. statistical quality control problem). For both, organized simplicities and unorganized complexities science has developed a number of analytical tools so we know how to tackle with these problems. Drawing upon technology history and develoment we can see that handling their complexity (static or detail complexity) depends on the reduction of problem components, thus their behavior becomes manageable and predictable.

Most interesting area extends between organized simplicities and unorganized complexities - this is the area of organized complexity. This area contains problems showing high dynamic complexity, even if the number of problem components is not high. As von Bertalanffy suggested, methodologies appropriate for the first two are useless in dealing with organized complexity. Most, if not all, economics and business problems fall in that category.

For organized complexity problems dynamic complexity yields behavior that can not be characterized by ordinary statistical tools. The problem behavior lies "in-between" rather than "within". This results in sometimes strikingly different conclusions than those generated by traditional forms of analysis, especially when what is being studied is dynamically complex or has a great deal of feedback from other sources, internal or external. As dynamic complexity stems from relationships existing among all problem components we need an approach focusing on those relationships instead of problem components and so far Systems Dynamics is the only one explicitly invoking relationships (feedback structures) as a focus.

6. Partitioning and Tearing Method

Problem systems can be best described by their structure and semantics. The structure, in addition to traditional descriptive way, can be represented by a graph or matrix showing which parts affect what other parts. The semantics concern is how the effects occur. Both do contribute to the understanding and explanation of a problem, and simple matrix calculus and algebra allow us to move beyond the limits of spoken language and advance to a higher degree of problem modeling. The method is called Partitioning and it is usually accompanied by its complement (Tearing Method) allowing to split complex linear equations systems into smaller chunks; both were proposed by G. Kron (1963). The Partitioning Method is using an ordinary graph ordering algorithm where vertices represent problem variables and relations between them become arcs.

The analysis of problem structure and behavior and resulting Problem Influence are in close proximity to other problem solving techniques that have been extensively used in recent years – "storytelling" and mental modeling. Both techniques, propagated by Learning Organization Center of Massachussetts Institute of Technology, see a problem not only as the discrepancy between "what is" and "what should be" but as the path linking decision maker with the future. D. Ingvar showed in 1985 (Ingvar 1985, p. 127–136) that while solving problems part of our mind remains connected with our foreseen future crafting a sequential time oriented graph where variables integrate our future to present problem thinking. Moreover, that graph is not simple and usually has many returns and dependencies on earlier events. Our "future memory" is activated by arising problems – otherwise remains dormant.

Storytelling, Influence Diagrams (result of using Partitioning Method) and similar have proved to be useful in practical decision making (Huff 1994). For problems containing dynamic complexity (feedback loops) our intuitional understanding is rarely sufficient; we need a complement from outside and Partitioning Method is a tool of invaluable importance for modeling dynamic problems. Partitioning Method was extensively used in solving extremely complex problems; e.g. managing schooling and medical services in Northern Australia (Warfield 1976) or Klein – Goldberger model of the United States economy (Klein, Goldberger 1955). The method assumes that:

> Structure of a problem is represented by the graph showing which parts affect which other parts.

> There is a path from vertex xi to xj if and only if the behavior of xi affects the behavior of xj.

> Semantics of the problem concerns the rules for the behavior of its parts and their effect on each other.

> Effect is represented by an arrow.

Partitioning Method is a simple matrix algorithm enabling us to divide any complex set of interrelated variables into a set of "blocks". A block is a set of variables among which a clear feedback loop does exist. The above statement means that for a blosk it is impossible to establish clear and understandable cause- effect relation and such effect is lost. Within each block there are paths in both directions from every vertex (problem variable) to every other vertex in the same block and between a vertex in one block and a vertex in another block there can be at most a path in one direction. Thus, blocks contain variables linked with feedbacks and they constitutes dynamic parts of a problem (all variables are simultanously depending on each other). There are no two way links between blocks – different blocks are linked with simple relation.

We explain the Partition Method using a simple example of a problem describing relationships among production process, production output, and market demand. The semantics of the problem is straightforward and probably easily understandable for any practitioner. What is difficult to conceive is a very complex behavior pattern generated by a set of simple feedback loops within the problem structure. Let's suppose that story told by one practitioner is as follows.

> *"Our production process depends largely on production capacity. The final product is transferred instantly to the market and – in most cases – is not stored. In our market niche we have never satisfied existing demand and cash inflow occurs immediately after the product reaches the market. We have never had problems with our customers; they pay in timely manner and we never had to request operational loan from the bank. Current sale helps to cover future production expenses. Given our market and demand, we are optimists. Market size justiies that opinion. We do not expect future problems – we will always sell our product."*

Step 1 – Inventory of variables

We begin with translating problem story into a set of variables that constitute the story. In all cases it is a subjective operation, although itis relatively easy to reach an agreement regarding variables participating in problem description. We discern here:

> production process > production volume > production capacity

> sale > revenue > production input

> current demand > future demand > market size

Step 2 – Construction of symmetrical matrix

In constructing symmetrical matrix of the problem all variable defined in Step 1 should be included in the same order in columns and rows od the matrix. All matrix cells are spaces for marking simple cause-effect relations existing between any pair of variables. Diagonal cell will represent relation between the same variable and will be excluded from the analysis. Fig. 3 shows resulting symmetrical matrix:

	production capacity	production process	production input	revenue	sale	production volume	current demand	future demand	market size	New number	Assigned
production capacity	●										
production process	X	●	X								
production input			●	X							
revenue				●	X						
sale					●	X	X	X			
production volume		X				●					
current demand						X	●		X		
future demand						X	X	●			
market size									●		

Fig. 3 Matrix representation of problem
Source: author's elaboration (for all following matrices).

	production capacity	production process	production input	revenue	sale	production volume	current demand	future demand	market size	New number	Assigned
production capacity	●									1	
production process	X	●	X								
production input			●	X							
revenue				●	X						
sale					●	X	X	X			
production volume		X				●					
current demand						X	●		X		
future demand						X	X	●			
market size									●	2	

Fig. 4 Ordering relations for variables not involved in feedback

"X" in the matrix represent perceived simple relations between variable, where columns are predecessors (variables influencing) and rows are successors (variables influenced). For instance, mark "X" in second row (production process) and first column (production capacity) tells that production capacity (predecessor) will immediately affect production volume (successor). If that matrix were ordered one, all marks X above the diagonal would have shown feedback loops existing in the matrix (problem).

There are special columns on the matrix right side; "new number" will assign to any variable its new consecutive number determining its position in future ordered matrix; "assigned" columns, in turn, will subordinate variables belonging to a feedback loop to a variable representing that loop. New number is given to a variable that no longer is affected by any other variable and can be excluded from the matrix without loosing problem structure semantics.

Step 3 - Ordering relations for variables not involved in feedback

Since a variable does not belong to any circuit, there is no mark "X" in that variable row. We found two such variables in our problem (production capacity and market size) and these variable receive first two consecutive number (1 and 2). After assigning new number to variables, they remaind beyond our concern (all marks in variables columns and rows are crossed/eliminated). Fig. 4 show matrix with these two variables arranged.

Step 4 - Ordering relations for variables not involved in feedback

If all remaining variables have preceding X marks, we choose one of them. Let's pick up "production process"; we see that "production process" has one predecessor ("production input") which - in turn - is preceded by "revenue". We continue backwards until we arrive at the same variable twice (beginning and end of the loop). If that occurs we choose one variable as representative for that loop and assign all remaining variables in the loop to this variable. In our case we found: production process - production input - revenue - sale - production volume - production process. We can see that "production process" appears in the loop twice - it opens and closes the first detected feedback loop (Fig. 5).

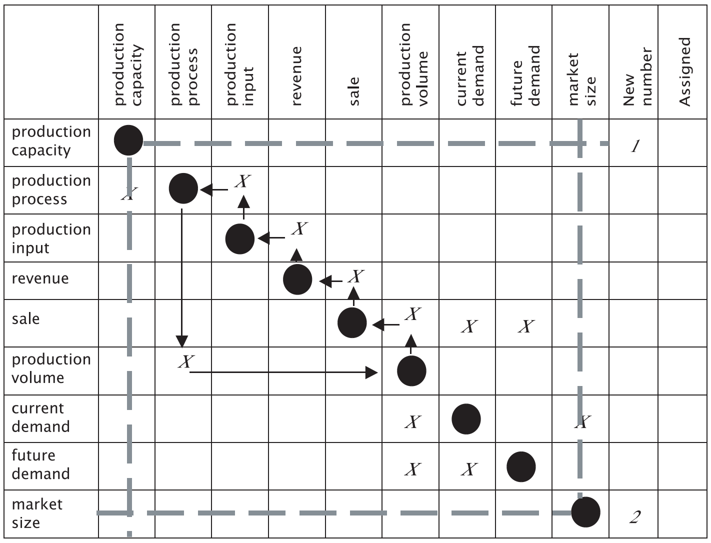


Fig. 5 Ordering relations for variables involved in feedback

Thus, we subordinate all variables in the loop to "production process" (see: column "Assignment" where subscripts 1 through 5 show the order of assignment) and cross all marks showing their involvement with other variables. This time, as opposed to variables not involved in feedback, we should respect relationship marks transferring them to respective variables denominated either as predecessors or successors (we use symbol ^ for transferred relations). Fig. 6 shows results of the operation.

There are still two variables showing predecessors; for "production process" previously transferred mark points to "future demand" as predecessor and for "future demand" "production demand" plays the same role. It is obvious that at the beginning of the process no one would argue that there is such direct relations between these two variables. Neither they did not exist in the original matrix nor were considered in the initial problem story. We perform exactly the same operation, this time only with two variables

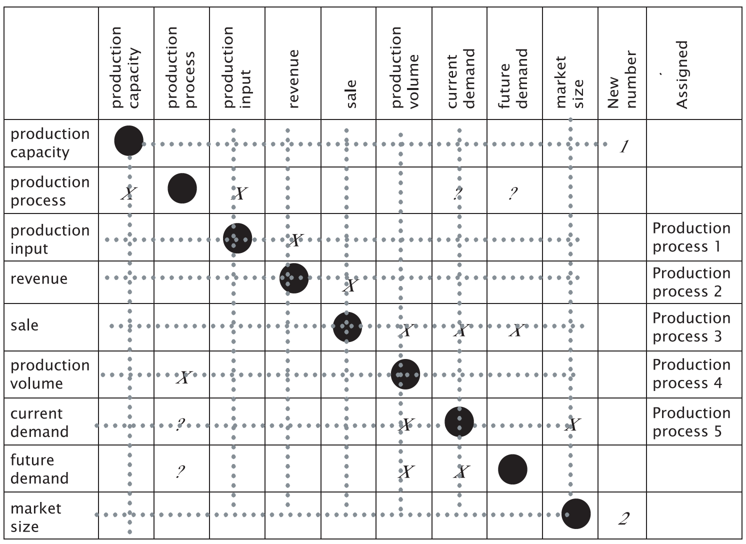


Fig. 6 Ordered matrix structure for "production process"

("production process" and "future demand"). After that operation "future demand" is subordinated to "production process" and all variables but one have been eliminated from the matrix. The variable "production process" is free, means that there is no predecessor affecting the variable. Whenever this happens we should assign a new consecutive number to that variable and all other assigned variables. In our case "production process" is receiving number 3 and all other variables receive following numbers in accordance to assigning them to the "production process".

	production capacity	production process	production input	revenue	sale	production volume	current demand	future demand	market size	New number	Assigned
production capacity	●									1	
production process	X	●	X				?	?		3	
production input			●	X						4	Production process 1
revenue				●	X					5	Production process 2
sale					●	X	X	X		6	Production process 3
production volume		X				●				7	Production process 4
current demand		?				X	●		X	8	Production process 5
future demand		?				X	X	●		9	Production process 6
market size									●	2	

Fig. 7 Partitioned matrix structure (without ordering)

Next step relies in re-writing the matrix, respecting all original relationships, with a new order determined by numbers assigned to all variables. That matrix is called ordered and all marks above the diagonal show real feedback loops existing in the matrix (problem structure). As it can be noted the matrix had originally 7 marks and after ordering there are only 6 of them. That means that our problem structure does contain 6 feedback loops with corresponding dynamic complexity. Fig. 7 and 8 shows the results.

Ordered matrix has a number of blocks. We identy blocks by analyzing variables along with their predecessors located above the diagonal for that variable. For variables "production capacity" and "market size" there are no such predecessors, therefore they will form single-variable blocks (independent variables in problem structure). All remaining variables have predecessors. Further blocks identification requires drawing within original matrix a set of symmetrical sub-matrices leaving no mark above the diagonal. For instance, block containing variable "production process" extends beyond that variable for if we close the block with that variable and "production input"only, a mark between "production input" and " revenue" would be left alone. Thus, we proceed until we identify another block containing all remaining variables. Fig. 9 presents all three blocks of our problem.

	production capacity	market size	production process	production input	revenue	sale	production volume	current demand	future demand
production capacity	●								
market size		●							
production process	X		●	X					
production input				●	X				
revenue					●	X			
sale						●	X	X	X
production volume			X				●		
current demand		X				X		●	
future demand						X		X	●

Fig. 8 Partitioned matrix structure (after ordering)

	production capacity	market size	production process	production input	revenue	sale	production volume	current demand	future demand
production capacity	●	1st block							
market size		●	2nd block						
production process	X		●	X					
production input				●	X				
revenue					●	X		3rd block	
sale						●	X	X	X
production volume			X				●		
current demand		X				X		●	
future demand						X		X	●

Fig. 9 Partitioned matrix structure (with blocks)

Third block has seven variables linked by feedback loops. We could present the structure of the problem without ordering these variables; the result of that, however, would be obscure and we would not be able to identify its internal structure. At this moment we may proceed with the analysis using the technique called Tearing Method. At this stage of the analysis we know that even with such a simple structure of the problem its behavior will show quite complicated pattern. Thus, it may be important to go further and order the 3rd block. We proceed as follows:

> Choose any variable in the block as starting variable.

> Trace its predecessor (preceding variable).

> Trace predecessor of that variable.

> Continue the process until we find return to starting variables.

> Remove all marks in identified path.

> Return to stage 1 and continue until all marks from above the diagonal are removed from the block.

Fig. 10 shows that operation; arrows point to the sequence of analyzed variables. Fig. 11 presents the structure of the problem based on partitioned and three times torn matrix.

	production process	production input	revenue	sale	production volume	current demand	future demand
production process	●	X					
production input		●	X				
revenue			●	X			
sale				●	X	X	X
production volume	X				●		
current demand				X		●	
future demand				X		X	●

Fig. 10. Analysis of block No. 3

7. Systems Dynamics

Sometimes we can estimate dynamic hypothesis directly through data experiments or even experiments in the real system. Most of the time, however, the conceptual model is so complex that its dynamic, behavioral impications are unclear. In other cases we are more interested in detail complexity (e.g. technical or statistical problems) where traditional simulation, aimed at numerical results instead of seeking for behavior pattern and displaying problem dynamics, is more appropriate. For organizaed complexity dynamics of the behavior is more important than numerical results of simulation. For those reasons Systems Dynamics modeling and simulation environment is far better option.

Feedback structures are responsible for most mistakes and fallacies made during problem solving and decision making processes. It has been repeatedly proven that while dealing with problems equipped with such a structure people continously mispercept and misjudged feedback impact on the problem (Sterman 2000, p. 26–27), reaching eventually solutions that ignore dynamic properties of the problem. Using Partitioning and Tearing methods, particularly for complex, dynamic problems, we reach the stage in problem modeling that allows us to use further procedure and simulate problem.

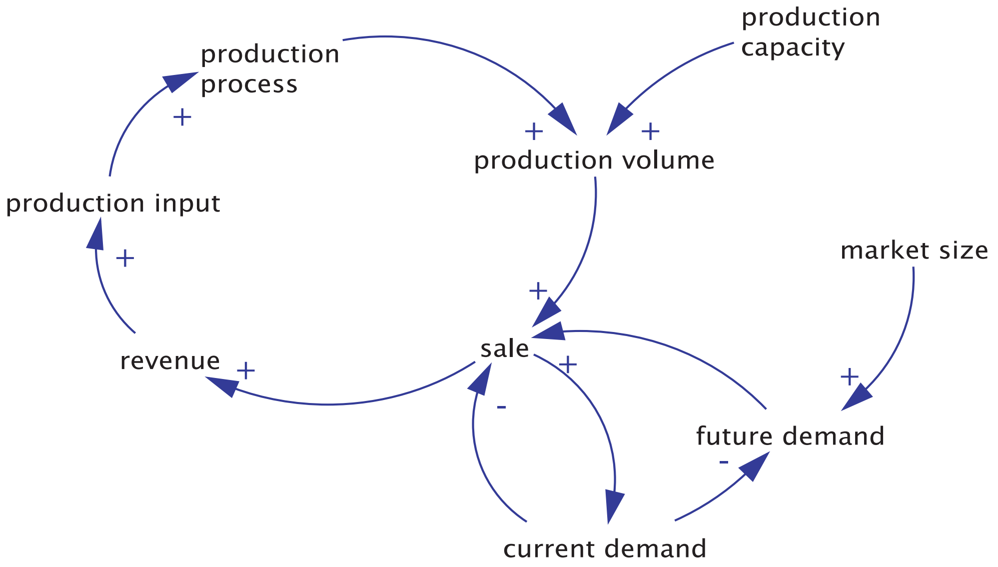


Fig. 11. Problem structure
Source: author's elaboration.

The creation of market and business patterns within Systems Dynamics framework possesses a set of special features which – for our purposes – can be reduced to the semantics and grammar of visual thinking. Semantics seeks best ways for representing the reality – accordance between the reality components and used symbols is a key issue here. It may be surprising that even for complex systems we do not need more that four generic construct that can reflect with sufficient precision behavioral properties of the components. They are:

> Stock variables; they change over time and stand for anything important for a model that accumulate and/or decrease their value. Comparing visual thinking to the structure of language, stock variables play the role of nouns. Stock variables are equipped with a "memory" (their present state depends, in addition to flow variables' influence, on their previous state) and are the best representations of a system. By default, stock variables are determined by differential calculus.

> Flow variables; they conduct external influences to and from stock variables. There is no other way of changing stock variables but through flow variables. Flow variables are like verbs describing changes in which stock variables participate. Flow variables assume values from accepted integration method.

> Converters; adjectives of the simulation language. They represent fixed (or arbitrarily predetermined) values existing in a model and they modify flow variables. Unlike stock and flow variables, converters can only conduct information (no physical values allowed)

> Links; they put together all other variables and close required feedback loops constituting the dynamics of a problem.

There are many available Systems Dynamics software packages enhancing our thinking capabilities. They all use the same or similar concepts and icons picturing above variables. We have chosen Vensim™ (by Ventana Systems) software; Fig. 12 present basic modeling resources (variables icons) used by software:

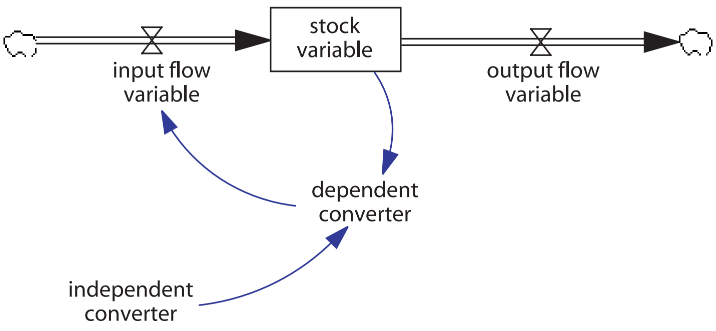


Fig. 12. Hypothetical use of modeling variables
Source: author's elaboration.

Let's go back to our El Farol problem. Having known how to use systems dynamics thinking tools and methodology, we do not have to wait and learn upon our experience (what ordinary customers must do). We can uncover bar attendance structure (visual thinking), build a graphical model, define mathematically or graphically its variables, simulate, and test problem behavior. We can learn without actually participating in the experiment and going/not going to the bar. Fig. 13 shows one possible structures for El Farol problem.

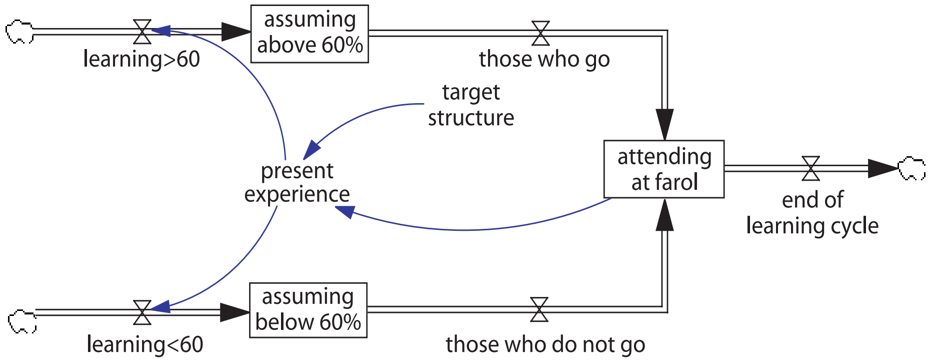


Fig. 13. El Farol dynamics in VENSIM™ environment
Source: author's elaboration.

Given the variety and richness of all available to El Farol customers' strategies, it would be impossible to understand its dynamic structure otherwise. Similarly, if we use classical interpretation of the bar dynamics (local rationality, maximizing strategies), our understanding would be incomplete at least and would not reveal the complexity of the problem. Using presented way of unleashing the El Farol (market) dynamics we can clearly identify patterns symptomatic for complex systems – butterfly effects, turbulent boundaries, transforming feedback loops, fractals, attractors, self-organization, and coupling. The market lives through El Farol case; the market lives through our mental model containing feedback structure as both show complex, non-linear dynamics.

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